INVESTIGATION OF SWIRLING FLOW OF A VISCOUS GAS NEAR THE STAGNATION LINE OF A BLUNT BODY

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The author examines the influence of rotation of the body and external swirl of the stream on flow of a viscous gas near the stagnation line of blunt axisymmetric bodies with permeable surfaces at low and medium Reynolds numbers. The study is made using the parabolized Navier-Stokes equations, which allow for the influence of the effects of molecular transfer in the entire compressed layer, including the region of transition through the shock wave. There is obtained a numerical solution to the problem over a wide range of change of the Reynolds number, the blowing parameter, and the other governing parameters. It has been shown that the presence of a nonzero component of the velocity vector in the circumferential direction in the shock layer can lead to a qualitative change of the flow character.

Axisymmetric swirling flows of liquid or gas are an important practical special case of three-dimensional flows in which all three components of the velocity vector differ from zero, but, because of the symmetry, the flow parameters depend only on two variables. This class of flows was investigated earlier only for quite large Reynolds numbers in the boundary layer model [1, 2] or in the thin shock layer model [3-5]. There are also papers (see, e.g.,[6-8]) in which the local similarity approximation of the Navier–Stokes equations has been applied to study nonswirling flows of a viscous gas.

1. Statement of the Problem. We consider uniform flow of a rarefied gas incident at zero angle of attack on an axisymmetric smooth blunt body with a permeable surface, rotating with angular velocity Ω_W^* about its axis. We postulate that in the vicinity of the symmetry axis in a rectangular coordinate system (x^1, x^2, x^3) (Ox^1 is the flow axis) the parameters of the oncoming stream satisfy the conditions

$$\operatorname{rot} V_\infty^* = (\Omega_\infty^*, 0, 0), \quad \rho_\infty^* = \text{const}, \quad P_\infty^* = P_{\infty 0}^* + \rho_\infty^* (\Omega_\infty^* r)^2/2 \tag{1.1}$$

($P_{\infty 0}^*$ is the gas pressure in the oncoming stream on the symmetry axis $r = 0$).

Taking into account that the Knudsen number is quite small, we shall study this flow in the framework of the Navier–Stokes equations. We choose a curvilinear coordinate system (ξ^*, η^*, ζ^*) as follows. We take the coordinate ζ^* along the normal to the wetted surface, η^* in the circumferential direction, and as the coordinate ξ^* we choose the central angle of a spherical coordinate system whose center coincides with that of a sphere touching the surface of the wetted body at the stagnation point. The solution of the system of Navier–Stokes equations near the stagnation line we represent in the form of expansions in power series in $\sin \xi^*$ and $\cos \xi^*$:

$$\begin{gathered} u^* = V_\infty^* (\sin \xi^* u_1(\zeta) + \ldots), \quad v^* = -V_\infty^* (\cos \xi^* v_0(\zeta) + \ldots), \\ W^* = V_\infty^* (\sin \xi^* W_1(\zeta) + \ldots), \quad \rho^* = \rho_\infty^* (\rho_0(\zeta) + \ldots), \\ T^* = (\gamma - 1) M_\infty^2 T_\infty^* (T_0(\zeta) + \ldots), \\ P^* = \rho_\infty^* V_\infty^{*2} (P_0(\zeta) + \sin^2 \xi^* P_2(\zeta) + \ldots), \\ \zeta^* = R\zeta, \quad \mu^* = \mu_\infty^* (\mu_0(\zeta) + \ldots). \end{gathered} \tag{1.2}$$

Here u^*, W^*, and v^* are the physical components of the velocity vector in the directions ξ^*, η^*, ζ^*, respectively; P^*, T^*, ρ^*, μ^* are the pressure, absolute temperature, density and viscosity of the gas; R is the body radius of curvature at the stagnation point; and V_∞^*, M_∞ are the absolute velocity and the Mach number of the incident flow at the stagnation point.

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If we substitute expansions (1.2) into the Navier–Stokes equations in which we have omitted terms which are negligible in the flow regimes considered [9], then, because of the ellipticity of the original problem, the system of equations for the coefficients of this expansion in any approximation contains unknown quantities from a higher approximation, and therefore the problem remains unclosed. To close it we apply the method of truncated series suggested in [6, 10]. Here the solutions in the first and second approximations differ insignificantly [10], and this approach is quite accurate. This is supported by the results of [11-14], where it is shown, by comparison with the solutions of two-dimensional problems and experimental data, that for the case of flow over a spherically blunted body the local similarity approximation leads to error mainly in the thickness of the perturbed region ahead of the blunt body. This error is about 10%, increases as M_∞ decreases, and depends slightly on Re. Since, as the analysis of [15] shows, the possibility of using the local similarity approximation to investigate flow on the stagnation line is determined by the parabolic degeneracy of the original problem and is associated with a thin shock layer, it is valid to use this model even to calculate swirling flows, since, other conditions being equal, the centrifugal forces arising here lead to greater spreading of the gas in the compressed layer and to a decrease of its total thickness.

Finally, the system of ordinary differential equations for the first terms of expansion (1.2), written in dimensionless form and transformed to Dorodnitsyn-type variables, has the form (we omit the subscripts 0 and 1)

$$z = 2\int_0^\zeta (1+\zeta)\,\rho\, d\zeta, \quad u = \frac{\partial\varphi}{\partial\zeta}, \quad w = \frac{W}{w_* \sqrt{\psi}}, \tag{1.3}$$

$$v = \frac{\varphi}{\psi}\psi', \quad l = \frac{2\mu}{\mathrm{Re}_\infty}\frac{\psi^{3/2}}{\psi'}, \quad \rho = \frac{1}{\psi'}, \quad \psi = (1+\zeta)^2,$$

$$p_2 = P_2 - \frac{\Omega^2}{2}\psi, \quad \Omega = \frac{\Omega_\infty^* R}{V_\infty^*}, \quad w_* = \begin{cases} \dfrac{\Omega_\infty^* R}{V_\infty^*} & (\Omega_\infty^* \neq 0), \\ \dfrac{\Omega_w^* R}{V_\infty^*} & (\Omega_\infty^* = 0); \end{cases}$$

$$(l\varphi'')' = p_2\psi' - \varphi\varphi'' + \frac{1}{2}(\varphi')^2 - \frac{1}{2}\frac{\psi'}{\psi}\varphi\varphi' - \frac{1}{2}\psi\left(w_*^2 w^2 - \Omega^2\psi'\right), \tag{1.4}$$

$$\left[l\sqrt{\psi}\left(w' + \frac{1}{2}\frac{\psi'}{\psi}w\right)\right]' = \left(\varphi'\sqrt{\psi} - \varphi\frac{\psi'}{\sqrt{\psi}}\right)w - \varphi\sqrt{\psi}\,w',$$

$$\left[\frac{4}{3}l\frac{\varphi}{\psi}\psi'' + \frac{\varphi^2}{\psi}\psi'\right]' = \frac{P\psi}{\psi'}\psi'' + \frac{\varphi\varphi'}{\psi}\psi' - \frac{\gamma-1}{\gamma}\frac{\psi}{\psi'}T',$$

$$\left(\frac{l}{\sigma}T'\right)' = -\frac{\varphi}{\gamma}T' - \frac{\gamma-1}{\gamma}\frac{\varphi\psi''}{\psi'}T - \frac{4}{3}l\left[\frac{(\varphi\psi')'}{\psi} - \frac{\varphi\psi'^2}{\psi^2}\right]^2,$$

$$p_2' = \frac{1}{2}\left(w_*^2 w^2 - \Omega^2\psi'\right) - \frac{\gamma-1}{2\gamma}\left[\frac{T'}{\psi'} - \frac{T\psi''}{\psi'^2}\right] + \frac{\varphi'^2}{2} + \frac{1}{2}\left(\frac{\varphi}{\psi}\right)^2\left(\psi'' - \frac{\psi'^2}{\psi}\right),$$

$$P = \frac{\gamma-1}{\gamma}\rho T, \quad \mu = \left[(\gamma-1)M_\infty^2 T\right]^\omega.$$

Here the primes denote derivatives with respect to z; σ = const is the Prandtl number; $\mathrm{Re}_\infty = \rho_\infty^* V_\infty^* R/\mu_\infty^*$; the volumetric viscosity coefficient is put equal to zero, and the coefficient of shear viscosity μ is proportional to the absolute temperature to the power ω.

The system of equations (1.4) must be solved with boundary conditions applied on the body surface and in the incident stream. Taking account of Eq. (1.1) and also neglecting the effects of slip and temperature jump on the body surface, which, as was shown in [16], are quantities of lower order for $\varepsilon = (\gamma - 1)/(\gamma + 1) \to 0$, we write these conditions in the variables (1.3) in the form

$$\varphi = (\rho v)_w = \varphi_w, \quad \varphi' = 0, \quad T = T_w, \quad \psi = 1, \quad w = \Omega_w/w_* \quad (z = 0); \tag{1.5}$$

$$\varphi' = 1, \quad \psi' = 1, \quad \psi = \varphi, \quad T = \left[(\gamma-1)M_\infty^2\right]^{-1}, \quad w = \Omega/w_*, \quad p_2 = 0 \quad (z \to \infty). \tag{1.6}$$

Equations (1.4) have special points [6-8, 17], one of which corresponds to the incident flow at infinity, and the other to the stagnation point located either on the body surface in the case of an impermeable wall [6, 17], or inside the flow (when blowing is present [7, 8]). As is shown by analysis, the presence of gas swirling in the flow does not affect the nature of the special points of Eq. (1.4).

2. Numerical Solution of the Problem. The boundary problem of Eqs. (1.4)-(1.6) was solved numerically using a finite difference scheme [18] with order of approximation $O(\Delta z^4)$. Here each of the equations of third order was reduced to a system of equations of first order. Then these equations were linearized in an appropriate manner and approximated by finite differences with an accuracy of $O(\Delta z^4)$. The system of difference equations thus obtained was solved in turn in the order in which Eq. (1.4) is written. A marching method was used in which the boundary conditions were transferred from the body surface to the external boundary. The next equation of Eq. (1.4) was integrated from the external boundary to the body surface by Simpson's Rule with the same accuracy. To make it easier to integrate all the equations with a single algorithm the second boundary condition for the function ψ at infinity ($\psi \to \varphi$) was transferred, analogously to [8], to the body surface by a single integration of the momentum equation in the normal projection from some point z_∞ located in the incident flow to the surface. The new boundary condition has the form

$$\frac{8}{3}\frac{\mu_w\varphi_w}{\mathrm{Re}_\infty}\psi_w'' - \psi_w'\left[\left(1+\frac{1}{\gamma M_\infty^2}\right)\varphi(z_\infty) - \varphi_w^2\psi_w' + \int_0^{z_\infty}\left(\varphi'\psi'\frac{\varphi}{\psi} + \frac{\gamma-1}{\gamma}T\right)dz\right] + \frac{\gamma-1}{\gamma}T_w = 0. \tag{2.1}$$

For a fixed difference mesh, as Re increases the zone of transition through the shock becomes less than the integration step size, leading to oscillations of the numerical solution. Therefore, at large Re ($\mathrm{Re}_\infty \geq 2.5\cdot10^3$), when the shock wave structure has only a slight effect on the flow characteristics in the shock layer, there was an artificial increase of the viscosity in the shock region, for which the position was determined by the behavior of ψ'. As a result the shock layer region contains not less than 3-4 cells of the difference mesh. In the boundary layer region the viscosity remained true, but at large Re there was bunching of the difference mesh there.

In all the calculations we assumed $\gamma = 1.4$, $\sigma = 0.71$, $\omega = 0.5$. The remaining parameters were varied over the following ranges:

$$10 \leqslant \mathrm{Re}_\infty \leqslant 2.5\cdot10^4,\ 0 \geqslant \varphi_w \geqslant -0{,}5,$$
$$0{,}05 \leqslant T_w \leqslant 02,\ 0 \leqslant \Omega \leqslant 2,\ 0 \leqslant \Omega_w \leqslant 2. \tag{2.2}$$

In the solution process we found profiles of the desired functions across the compressed layer, and also the coefficients of friction and heat transfer on the body surface:

$$\tau = \mu\frac{du}{d\zeta}\frac{\sqrt{\mathrm{Re}}}{\mathrm{Re}_\infty},\quad q = \mu\frac{dT}{d\zeta}\frac{\sqrt{\mathrm{Re}}}{\sigma\,\mathrm{Re}_\infty},\quad \mathrm{Re} = \frac{\rho_\infty^* V_\infty^* R}{\mu^*(T_0^*)}. \tag{2.3}$$

3. Discussion of the Results. We first address the question of applicability of the uniform gas model to calculate real flows about blunt bodies. Generally speaking, in flow over bodies dissociation and ionization reactions can proceed in the perturbed flow regions. However, it follows from [17] that there is quite a large range of altitudes, flight speeds and body sizes for which the characteristic flow Damkeller number is small, and therefore the chemical reactions can be considered as frozen, and the uniform gas model is applicable with sufficient accuracy. It is known also that the presence of chemical reactions in the flow has only a slight influence on such flow characteristics as the pressure distribution. In addition, the uniform gas model with an effective adiabatic index γ is widely used in the literature to describe flows in chemical equilibrium. In these situations the results obtained with the uniform gas model retain their practical value and can be used to model the real flows.

Figures 1-4 shows some of the computed results. Figure 1 shows profiles of u (lines 1, 3), w (lines 2, 4) and T (lines 5, 6) across the compressed layer for $T_W = 0.15$, $\varphi_w = 0$ for different swirl values of the incident flow ($\Omega = 0.75$; 1.375 – a, b) and Reynolds number ($\mathrm{Re}_\infty = 100$, lines 3-5, $\mathrm{Re}_\infty = 10^3$, lines 1, 2 and 6). It can be seen that for $\mathrm{Re}_\infty = 100$ the shock wave is quite smeared, and that even for $\mathrm{Re}_\infty = 10^3$ the flow goes into the viscous shock layer regime and the thickness of the shock layer is 2-3% of that of the entire com-

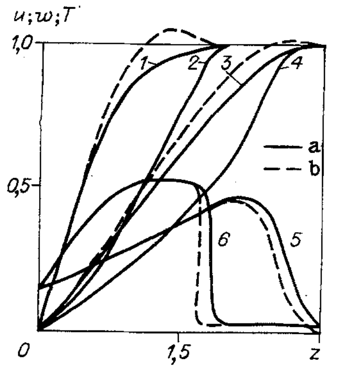

Fig. 1

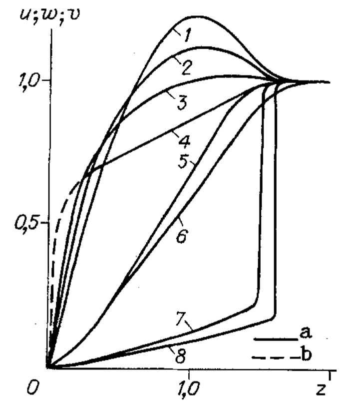

Fig. 2

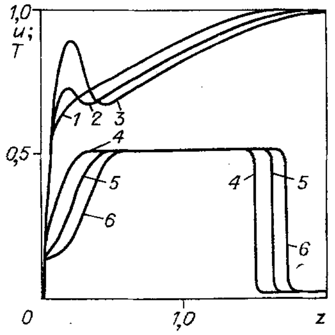

Fig. 3

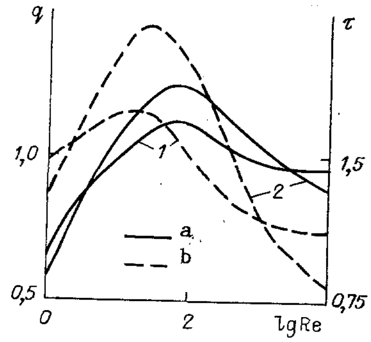

Fig. 4

pressed layer. As is shown by the computations, the presence of gas swirl has quite a strong influence on the nature of the flow in the perturbed region. This can be seen clearly in Fig. 2, which shows profiles of u (lines 1-4), w (5-6) and v (7-8) across the boundary layer on an impermeable surface for $T_w = 0.15$ and $Re_\infty = 10^4$, $2.5 \cdot 10^4$ (a, b) and various values of the swirl parameter of the incident flow ($\Omega = 0.01$ for lines 4, 6, 8, $\Omega = 0.875$ for 3, $\Omega = 1.25$ for 2, $\Omega = 1.6$ for 1, 5, and 7). As Ω increases the u profile loses its monotonic nature and acquires the character of a maximum near the shock. The appearance of a maximum is explained by the fact that the flow is strongly influenced in this region by centrifugal forces which become considerable at that point of the compressed layer and are not equal to the pressure gradient forces.

For fixed Ω the value of this maximum increases with increase of Re and tends to some finite limit, which agrees well with the asymptotic analysis of the boundary problem of Eqs. (1.4)-(1.6) for $Re_\infty \to \infty$. As Re_∞ decreases the influence of the effects of molecular transport become appreciable over the entire perturbed flow region, the maximum decreases, and, beginning at a certain Re_∞^*, it disappears. In contrast with the tangential component of the velocity vector the influence of Ω on the profile of the circumferential velocity component is less apparent. We note also that, other conditions being equal, an increase of Ω leads to a decrease of the standoff distance of the shock from the body surface.

Analysis of the computations has shown that if the wetted body is rotated, in addition to swirl of the incident flow, then the flow structure becomes even more complex. In particular, for large enough Re ($Re_\infty \gtrsim 2 \cdot 10^3$) the influence of body rotation is localized in the boundary layer near the body surface, and consequently the u profile near the body may have additionally two local extrema — a maximum and a minimum, while the profile of the circumferential velocity component has a characteristic minimum. As Re decreases the local extrema in the u profiles near the surface disappear, and the minimum of w increases and moves to the center of the shock layer.

Blowing of gas from the body surface leads to nonlinear effects of interaction with the swirling flow in the compressed layer. A typical example of the dependence of the flow structure on the blowing parameter is shown in Fig. 3, which gives profiles of u (lines 1-3) and T (lines 4-6) across the layer for $Re_\infty = 10^4$, $\Omega = 0$, $\Omega_w = 2.0$ for various values of φ_w ($\varphi_w = 0$ for lines 1, 4, $\varphi_w = -0.1$ for 2, 5, $\varphi_w = -0.2$ for 3, 6). It can be seen

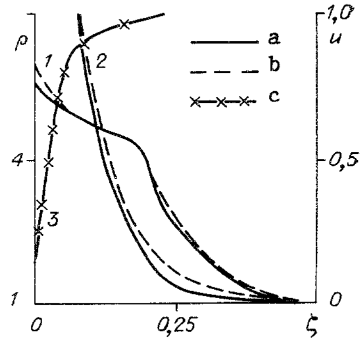

Fig. 5

that as the gas flow rate through the surface increases (other conditions being equal) the u profile ceases to be monotonic and acquires characteristic extrema at the boundaries of the mixing layer which is formed between the two inviscid layers adjacent to the body surface and the shock wave.

It follows from the computed results that at small Re blowing with flow rates $-\varphi_w \leqslant 0.15$ has little effect on the nature of flow in the compressed layer, and on the heat flux and the friction coefficient on the body surface. As Re increases, for fixed gas flow rate through the surface, the influence of blowing increases – the shock standoff distance increases, and near the body surface a layer of blown gas is clearly visible. On the whole, the analysis shows that, as was true for two-dimensional flows [7], with regard to the characteristic of the blowing intensity and its influence on the flow structure one should not use the relative mass flow rate of gas through the surface φ_w, but the blowing parameter, usually applied in the theory of boundary layers and viscous shock layers [19], $F_W = -\varphi_w \sqrt{Re/2}[(\gamma - 1)/2\gamma]^{1/4}(-P_{2w})^{-1/4}$. In particular, the numerical results support the conclusion, made in [9] on the basis of an asymptotic analysis, that at large Re and $F_W >> 1$ the entire perturbed flow region can be divided into four sublayers: the transition region through the shock layer, the inviscid shock layer and the layer of blown gas, adjacent, respectively, to the shock wave and the body surface, and also a mixing layer located between them. And while we can neglect the effects of molecular transport to a first approximation in the inviscid shock layer and the blown gas layer, these effects play the main role in the mixing layer and in the shock wave.

Figure 4 shows the influence of flow swirl on the dependence on Re of the heat transfer coefficient q and the friction factor τ (lines a, b) on the body surface, for $T_W = 0.1$, $\varphi_w = 0$, $\Omega_W = 0$, $\Omega = 0$; 1.5 (lines, 1, 2). It can be seen that these distributions have a characteristic local maximum, and for τ the maximum is more strongly pronounced, and its location is displaced towards smaller Re. Other conditions being equal, swirl of the incident flow and rotation of the body increase this local maximum in the distributions of τ and q in comparison with the cases $\Omega = \Omega_W = 0$. The position of this maximum is practically independent of Ω and Ω_W, but while an increase of Ω_W leads to an increase of τ and q over the entire Re range, an increase of Ω increases τ and q for $0.5 \lesssim \log Re \lesssim 3.0$ and decreases it for $\log Re \lesssim 0.5$ and $\log Re \gtrsim 3.0$.

It should be noted that an increased swirl of the incident flow for $\Omega_W = 0$ results in τ and q going to their boundary layer values for large enough Re compared with the case $\Omega = 0$. For example, for $\Omega = 0$ the difference between the value of τ, computed for Re = 10^3 and 10^4 is 6%, and for $\Omega = 1.5$ it is 35%. For the heat transfer coefficients these differences are, respectively –2 and 18%. Thus, for large enough swirl of the incident flow, for correct construction of the asymptote of the Navier–Stokes equations for large Re one should take account of viscous-inviscid interaction, which is not accounted for in the usual formulation of the problem in the first approximation of boundary layer theory.

In conclusion we compare the numerical results obtained by this method with some results obtained using the complete [12] and the parabolized Navier–Stokes equations [20]. Figure 5 shows profiles of the density across the compressed layer for $Re_\infty = 30$, $M_\infty = 4.2$, $\gamma = 1.4$, $\Omega_W = \Omega = 0$, $\varphi_w = 0$, $T_W = 0.26$; 1.01 (lines 1, 2) and of velocity u for $Re_\infty = 700$, $\gamma = 5/3$, $T_W = 0.03$, $\Omega_W = \Omega = \varphi_w = 0$ (line 3). Here "a" is results of this work, and "b" and "c" are from [12] and [20], respectively. The comparison shows quite satisfactory agreement.

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